b) Montrer que $\int_0^{\frac{\pi}{4}} f(x)\,dx + \int_0^{\frac{\pi}{4}} f^2(x)dx = \frac{1}{3}$.

$$\int_0^{\frac{\pi}{4}} f(u)\,du + \int_0^{\frac{\pi}{4}} f^2(x)\,du$$

$$= \int_0^{\frac{\pi}{4}} \tan^2(x)\,dx + \int_0^{\frac{\pi}{4}} \tan^4(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} \left(\tan^2(x) + \tan^4(x)\right) dx$$

$$= \int_0^{\frac{\pi}{4}} \left(1 + \tan^2(x)\right) \cdot \tan^2(x)\,dx$$

$u'(x)$. $u^2(x)$

$u'u^n \longrightarrow \frac{u^{n+1}}{n+1}$

$$= \left[\frac{\tan^3(x)}{3}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\tan^3\left(\frac{\pi}{4}\right)}{3} - 0$$

$$= \boxed{\frac{1}{3}}$$